# Exercise 36

(a) Sketch the vector field $\mathbf{F}(x, y) = \mathbf{i} + x\,\mathbf{j}$ and then sketch some flow lines. What shape do these flow lines appear to have?

(b) If parametric equations of the flow lines are $x = x(t)$, $y = y(t)$, what differential equations do these functions satisfy? Deduce that $dy/dx = x$.

(c) If a particle starts at the origin in the velocity field given by $\mathbf{F}$, find an equation of the path it follows.

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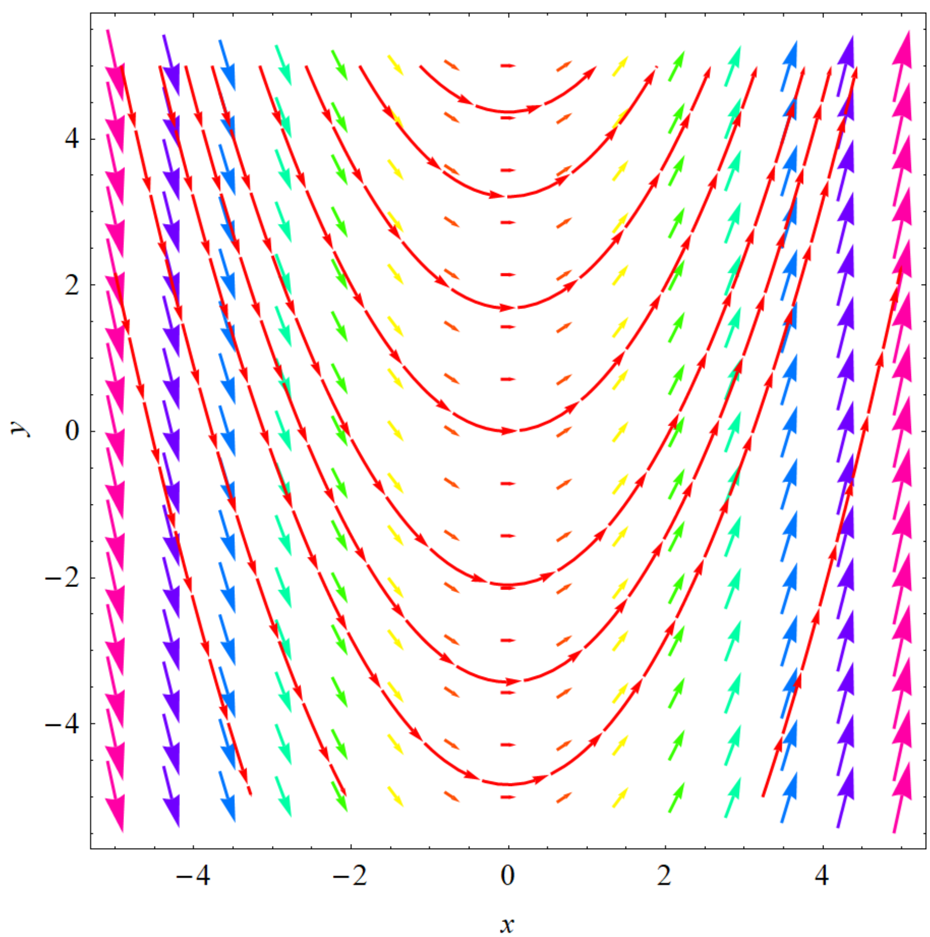
## Solution

The vector field $\mathbf{F}(x, y) = \mathbf{i} + x\,\mathbf{j}$ is shown below along with several flow lines in red.




Each of the vectors in the field is tangent to a possible flow line, similar to the way a velocity vector is tangent to the corresponding position vector.

$$\mathbf{F}(x(t), y(t)) = \frac{d\mathbf{X}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$

Set $dx/dt$ equal to the $x$-component of $\mathbf{F}$ and set $dy/dt$ equal to the $y$-component of $\mathbf{F}$.

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = x$$

From this first ODE, $dx = dt$, which makes the second one

$$\frac{dy}{dx} = x.$$

Integrate both sides with respect to $x$.

$$y(x) = \frac{1}{2}x^2 + C$$

Use the fact that the flow line has to go through $(0, 0)$ to determine $C$.

$$0 = \frac{1}{2}(0)^2 + C \quad \rightarrow \quad C = 0$$

Therefore,

$$y(x) = \frac{1}{2}x^2.$$